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**LETTER TO THE EDITOR**

# Highest weight representations of the quantum algebra $U_h(gl_\infty)$

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**Abstract.** A class of highest-weight irreducible representations of the quantum algebra $U_h(gl_\infty)$ is constructed, which is considerably larger than the currently known representations (Levendorskii S and Soibelman Y 1991 *Commun. Math. Phys.* **140** 399). Within each module a basis is introduced and the transformation relations of the basis under the action of the Chevalley generators are explicitly written.

The Lie algebra $gl_\infty$ and its completion and central extension $A_\infty$ [1, 2] play an important role in several branches of mathematics and physics. These algebras are of interest as examples of Kac–Moody Lie algebras of infinite type [1, 3–5]. They have applications in the theory of nonlinear equations [6], string theory, two-dimensional statistical models [7]. One of us (TP) studies new quantum statistics, based on the above algebras (see *Example 2* in [8] and the references therein), leading to local currents [9] for what are called $A$-spinor fields. It is natural to expect that the deformations of these algebras and their representations may also prove useful.

The quantum analogues of $gl_\infty$ and $A_\infty$ in the sense of Drinfeld [10], namely $U_h(gl_\infty)$ and $U_h(A_\infty)$, were worked out by Levendorskii and Soibelman [11]. These authors have constructed a class of highest weight irreducible representations, labelled by an arbitrary integer $s$.

In the present note we announce results on certain highest weight irreducible representations (irreps) of $U_h(gl_\infty)$. The $U_h(gl_\infty)$-modules, we study, are labelled by all possible complex sequences (see the notation below)

$$\{M\} \equiv \{M_i\}_{i\in\mathbb{Z}} \in \mathbb{C}^\infty \qquad \text{such that } M_i - M_j \in \mathbb{Z}_+ \qquad \forall i < j. \tag{1}$$

The signatures of the $U_h(gl_\infty)$-modules of Levendorskii and Soibelman [11] consist of all those sequences $\{M^{(s)}\}$, $s \in \mathbb{Z}$, from (1), for which $M_i^{(s)} = 1$, if $i < s$ and $M_i^{(s)} = 0$ for $i \geqslant s$.

Throughout the letter we use the notation (most of them standard): $\mathbb{Z}$, $\mathbb{N}$, $\mathbb{Z}_+$, $\mathbb{C}$—all integers, positive integers, non-negative integers and complex numbers, respectively; $\mathbb{C}[[h]]$—the ring of all formal power series in $h$ over $\mathbb{C}$;

$$q = \mathrm{e}^{h/2} \in \mathbb{C}[[h]] \qquad [x] = \frac{q^x - q^{-x}}{q - q^{-1}} \in \mathbb{C}[[h]]. \tag{2}$$

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*The algebra* $U_h(gl_\infty)$. Following [11] we recall that $U_h(gl_\infty)$ is the Hopf algebra, which is a topologically free module over $\mathbb{C}[[h]]$ (complete in $h$-adic topology), with generators $\{E_i, F_i, H_i\}_{i\in\mathbb{Z}}$, the Chevalley generators and

1. Cartan relations:

$$\begin{aligned}
&[H_i, H_j] = 0\\
&[H_i, E_j] = (\delta_{ij} - \delta_{i,j+1})E_j\\
&[H_i, F_j] = -(\delta_{ij} - \delta_{i,j+1})F_j\\
&[E_i, F_j] = \delta_{ij}\frac{q^{H_i-H_{i+1}} - q^{-H_i+H_{i+1}}}{q-q^{-1}}.
\end{aligned} \tag{3}$$

2. $E$-Serre relations:

$$\begin{aligned}
&E_iE_j = E_jE_i \qquad \text{if } |i-j| \neq 1\\
&E_i^2E_{i+1} - (q+q^{-1})E_iE_{i+1}E_i + E_{i+1}E_i^2 = 0\\
&E_{i+1}^2E_i - (q+q^{-1})E_{i+1}E_iE_{i+1} + E_iE_{i+1}^2 = 0.
\end{aligned} \tag{4}$$

3. $F$-Serre relations:

$$\begin{aligned}
&F_iF_j = F_jF_i \qquad \text{if } |i-j| \neq 1\\
&F_i^2F_{i+1} - (q+q^{-1})F_iF_{i+1}F_i + F_{i+1}F_i^2 = 0\\
&F_{i+1}^2F_i - (q+q^{-1})F_{i+1}F_iF_{i+1} + F_iF_{i+1}^2 = 0.
\end{aligned} \tag{5}$$

A co-unity $\varepsilon$, a co-multiplication $\Delta$ and an antipode $S$ are defined as:

$$\begin{aligned}
&\varepsilon(E_i) = \varepsilon(F_i) = \varepsilon(H_i) = 0\\
&\Delta(H_i) = H_i \otimes 1 + 1 \otimes H_i\\
&\Delta(E_i) = E_i \otimes q^{(H_i-H_{i+1})/2} + q^{(-H_i+H_{i+1})/2} \otimes E_i\\
&\Delta(F_i) = F_i \otimes q^{(H_i-H_{i+1})/2} + q^{(-H_i+H_{i+1})/2} \otimes F_i\\
&S(H_i) = -H_i \qquad S(E_i) = -qE_i \qquad S(F_i) = -q^{-1}F_i.
\end{aligned} \tag{6}$$

The tensor products are topological throughout, namely the algebraic tensor products are replaced with their completion in the $h$-adic topology.

Note that $\{E_i, F_i, H_i\}_{i\in\mathbb{N}}$ generate a Hopf subalgebra $U_h(gl_0(\infty))$ of $U_h(gl_\infty)$.

*Representations of* $U_h(gl_\infty)$. Let $W$ be a topologically free $\mathbb{C}[[h]]$-module. We recall that a $\mathbb{C}[[h]]$-homomorphism $\rho : U_h(gl_\infty) \to \text{End } W$ is a representation of $U_h(gl_\infty)$ in $W$ (equivalently, $W$ is a $U_h(gl_\infty)$-module) provided $\rho$ is continuous in the $h$-adic topology.

We proceed to define the $U_h(gl_\infty)$-module $V(\{M\})$ with a highest weight $\{M\}$ and its (topological) basis. The basis $\Gamma(\{M\})$, called a central basis ($C$-bases) [12], consists of all $C$-patterns

$$(M) \equiv \left[\begin{array}{cccccccc}
\ldots & M_{1-\theta-k} & \ldots & M_{-1} & M_0 & M_1 & \ldots & M_{k+\theta-1}, \ldots\\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots\\
 & M_{1-\theta-k,2k+\theta-1} & \ldots & M_{-1,2k+\theta-1} & M_{0,2k+\theta-1} & M_{1,2k+\theta-1} & \ldots & M_{k+\theta-1,2k+\theta-1}\\
 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \\
 & & & M_{-1,3} & M_{03} & M_{13} & & \\
 & & & M_{-1,2} & M_{02} & & & \\
 & & & & M_{01} & & &
\end{array}\right] \tag{7}$$

where $k \in \mathbb{N}, \theta = 0, 1$. Each such pattern is an ordered collection of complex numbers:

$$M_{i,2k+\theta-1} \qquad \forall k \in \mathbb{N}$$
$$\theta = 0, 1 \; i \in [-\theta - k + 1, k - 1] \equiv \{-\theta - k + 1, -\theta - k + 2, \ldots, k - 1\} \tag{8}$$

which satisfy the conditions:

(i) there exist a positive, depending on $(M)$, integer $N_{(M)}$ such that

$$M_{i,2k+\theta-1} = M_i \; \forall k > N_{(M)} \qquad \theta = 0, 1 \; i \in [-\theta - k + 1, k - 1] \tag{9}$$

(ii)

$$M_{i+\theta-1,2k+\theta} - M_{i,2k+\theta-1} \in \mathbb{Z}_+ \qquad M_{i,2k+\theta-1} - M_{i+\theta,2k+\theta} \in \mathbb{Z}_+ \; \forall k \in \mathbb{N}$$
$$\theta = 0, 1 \; i \in [1 - \theta - k, k - 1]. \tag{10}$$

Denote by $V_0(\{M\})$ the free $\mathbb{C}[[h]]$-module with generators $\Gamma(\{M\})$ and let $V(\{M\})$ be its completion in the $h$-adic topology. $V(\{M\})$ is a topologically free $\mathbb{C}[[h]]$-module with (topological) basis $\Gamma(\{M\})$. It consists of all formal power series in $h$ with coefficients in $V_0(M)$, i.e. for any $v \in V(M)$

$$v = v_0 + v_1 h + v_2 h^2 + \cdots \qquad (v_0, v_1, v_2, \ldots \in V_0(M)). \tag{11}$$

End $V(\{M\})$ is a $\mathbb{C}[[h]]$-module, consisting of all $\mathbb{C}[[h]]$-linear maps of $V(\{M\})$. If $\varphi$ is a $\mathbb{C}[[h]]$-linear map in $V_0(\{M\})$, we extend it to a continuous linear map on $V(\{M\})$ setting $\varphi v = \varphi v_0 + (\varphi v_1)h + (\varphi v_2)h^2 + \cdots$. Therefore the transformation of $V(\{M\})$ under the action of $\varphi$ is completely defined, if $\varphi$ is defined on $\Gamma(\{M\})$.

We pass to turn $V(\{M\})$ into a $U_h(gl_\infty)$ module. To this end we first introduce some appropriate notation [12]. Denote by $(M)_{\pm\{j,p\}}$ and $(M)^{\pm\{l,q\}}_{\pm\{j,p\}}$ the patterns obtained from the $C$-pattern $(M)$ (7) after the replacements

$$M_{lq} \to M_{lq} \pm 1 \qquad M_{jp} \to M_{jp} \pm 1 \tag{12}$$

correspondingly, and let

$$S(j, l; \nu) = \begin{cases} (-1)^\nu & \text{for } j = l \\ 1 & \text{for } j < l \\ -1 & \text{for } j > l \end{cases} \qquad \theta(i) = \begin{cases} 1 & \text{for } i \geqslant 0 \\ 0 & \text{for } i < 0 \end{cases} \qquad L_{ij} = M_{ij} - i. \tag{13}$$

Set moreover

$$E_i^0 = F_i \qquad E_i^1 = E_i \qquad i \in \mathbb{Z}. \tag{14}$$

Let $\{\rho(E_i),\ \rho(F_i),\ \rho(H_i)\}_{i\in\mathbb{Z}}$ be a collection of $\mathbb{C}[[h]]$-endomorphisms of $V(\{M\})$, defined on any $C$-pattern $(M) \in \Gamma(\{M\})$, as follows:

$$\rho(E_{-1}^{1-\mu})(M) = ([L_{-1,2} - L_{0,1} - \mu][L_{0,1} - L_{0,2} + \mu])^{1/2}(M)_{-(-1)^\mu\{0,1\}} \qquad \mu = 0, 1 \tag{15}$$

$$\begin{aligned}
\rho(E^{\mu}_{(-1)^\nu i-1})(M) = &- \sum_{j=1-i-\nu}^{i-1} \sum_{l=-i}^{i+\nu-1} S(j, l; \nu)\Bigg( - \prod_{k\neq l=-i}^{i+\nu-1} [L_{k,2i+\nu} - L_{j,2i+\nu-1} - (-1)^\nu \mu] \\
&\times \prod_{k=1-i}^{i+\nu-2} [L_{k,2i+\nu-2} - L_{j,2i+\nu-1} - (-1)^\nu \mu] \\
&\times \Bigg( \prod_{k\neq j=1-i-\nu}^{i-1} [L_{k,2i+\nu-1} - L_{j,2i+\nu-1}][L_{k,2i+\nu-1} - L_{j,2i+\nu-1} + (-1)^{\mu+\nu}] \Bigg)^{-1}
\end{aligned}$$

$$\times\left(\prod_{k=-i-\nu}^{i}[L_{k,2i+\nu+1}-L_{l,2i+\nu}+(-1)^{\nu}(1-\mu)]\right.$$
$$\left.\times\prod_{k\neq j=1-i-\nu}^{i-1}[L_{k,2i+\nu-1}-L_{l,2i+\nu}+(-1)^{\nu}(1-\mu)]\right)$$
$$\times\left(\prod_{k\neq l=-i}^{i+\nu-1}[L_{k,2i+\nu}-L_{l,2i+\nu}][L_{k,2i+\nu}-L_{l,2i+\nu}+(-1)^{\mu+\nu}]\right)^{-1}\Bigg)^{1/2}$$
$$\times(M)^{-(-1)^{\mu+\nu}\{l,2i+\nu\}}_{-(-1)^{\mu+\nu}\{j,2i-1+\nu\}}\qquad i\in\mathbb{N}\ \mu,\nu=0,1 \tag{16}$$

$$\rho(H_i)(M)=\left(\sum_{j=-|i|}^{|i|+\theta(i)-1}M_{j,2|i|+\theta(i)}-\sum_{j=-|i|+1-\theta(i)}^{|i|-1}M_{j,2|i|+\theta(i)-1}\right)(M)\qquad i\in\mathbb{Z}. \tag{17}$$

If a pattern from the right-hand side of (16) does not belong to $\Gamma(\{M\})$, i.e. it is not a $C$-pattern, then the corresponding term has to be deleted. (The coefficients in front of all such patterns are undefined, they contain zero multiples in the denominators. Therefore an equivalent statement is that all terms with zeros in the denominators have to be removed.) With this convention all coefficients in front of the $C$-patterns in right-hand side of (15)–(17) are well defined as elements from $\mathbb{C}[[h]]$.

*Proposition 1.* The endomorphisms $\{\rho(E_i),\rho(F_i),\rho(H_i)\}_{i\in\mathbb{Z}}$ satisfy (3)–(5) with $\rho(E_i),\rho(F_i),\rho(H_i)$ substituted for $E_i, F_i, H_i$, respectively.

The proof is extremely lengthy. We only mention that as an intermediate step one has to use such non-trivial $q$-identities as:

$$\sum_{j=1-k}^{k-1}\sum_{l=-k}^{k-1}\left(\prod_{i\neq l=-k}^{k-1}[L_{i,2k}-L_{j,2k-1}-1]\prod_{i=1-k}^{k-2}[L_{i,2k-2}-L_{j,2k-1}-1]\right.$$
$$\left.\times\prod_{i=-k}^{k}[L_{i,2k+1}-L_{l,2k}]\prod_{i\neq j=1-k}^{k-1}[L_{i,2k-1}-L_{l,2k}]\right)$$
$$\times\left(\prod_{i\neq j=1-k}^{k-1}[L_{i,2k-1}-L_{j,2k-1}][L_{i,2k-1}-L_{j,2k-1}-1]\right.$$
$$\left.\times\prod_{i\neq l=-k}^{k-1}[L_{i,2k}-L_{l,2k}][L_{i,2k}-L_{l,2k}-1]\right)^{-1}$$
$$-\sum_{j=1-k}^{k-1}\sum_{l=-k}^{k-1}\left(\prod_{i\neq l=-k}^{k-1}[L_{i,2k}-L_{j,2k-1}]\prod_{i=1-k}^{k-2}[L_{i,2k-2}-L_{j,2k-1}]\right.$$
$$\left.\times\prod_{i=-k}^{k}[L_{i,2k+1}-L_{l,2k}+1]\prod_{i\neq j=1-k}^{k-1}[L_{i,2k-1}-L_{l,2k}+1]\right)$$
$$\times\left(\prod_{i\neq j=1-k}^{k-1}[L_{i,2k-1}-L_{j,2k-1}][L_{i,2k-1}-L_{j,2k-1}+1]\right.$$
$$\left.\times\prod_{i\neq l=-k}^{k-1}[L_{i,2k}-L_{l,2k}][L_{i,2k}-L_{l,2k}+1]\right)^{-1}$$
$$=\left[\sum_{j=-k+1}^{k-1}L_{j,2k-1}-\sum_{j=-k+1}^{k-2}L_{j,2k-2}-\sum_{j=-k}^{k}L_{j,2k+1}+\sum_{j=-k}^{k-1}L_{j,2k}-1\right]\quad k\in\mathbb{N} \tag{18}$$

$$\sum_{j=-k}^{k-1}\sum_{l=-k}^{k}\Bigg(\prod_{i\neq l=-k}^{k}[L_{i,2k+1}-L_{j,2k}+1]\prod_{i=1-k}^{k-1}[L_{i,2k-1}-L_{j,2k}+1]$$
$$\times\prod_{i=-k-1}^{k}[L_{i,2k+2}-L_{l,2k+1}]\prod_{i\neq j=-k}^{k-1}[L_{i,2k}-L_{l,2k+1}]\Bigg)$$
$$\times\Bigg(\prod_{i\neq j=-k}^{k-1}[L_{i,2k}-L_{j,2k}][L_{i,2k}-L_{j,2k}+1]$$
$$\times\prod_{i\neq l=-k}^{k}[L_{i,2k+1}-L_{l,2k+1}][L_{i,2k+1}-L_{l,2k+1}+1]\Bigg)^{-1}$$
$$-\sum_{j=-k}^{k-1}\sum_{l=-k}^{k}\Bigg(\prod_{i\neq l=-k}^{k}[L_{i,2k+1}-L_{j,2k}]\prod_{i=1-k}^{k-1}[L_{i,2k-1}-L_{j,2k}]$$
$$\times\prod_{i=-k-1}^{k}[L_{i,2k+2}-L_{l,2k+1}-1]\prod_{i\neq j=-k}^{k-1}[L_{i,2k}-L_{l,2k+1}-1]\Bigg)$$
$$\times\Bigg(\prod_{i\neq j=-k}^{k-1}[L_{i,2k}-L_{j,2k}][L_{i,2k}-L_{j,2k}-1]$$
$$\times\prod_{i\neq l=-k}^{k}[L_{i,2k+1}-L_{l,2k+1}][L_{i,2k+1}-L_{l,2k+1}-1]\Bigg)^{-1}$$
$$=\Bigg[\sum_{j=-k-1}^{k}L_{j,2k+2}-\sum_{j=-k}^{k}L_{j,2k+1}-\sum_{j=-k}^{k-1}L_{j,2k}+\sum_{j=-k+1}^{k-1}L_{j,2k-1}-1\Bigg]\quad k\in\mathbb{N}. \tag{19}$$

The proof of the above identities, which are of independent interest, together with the complete proof of the proposition will be given elsewhere.

A (topological) basis $\{e_n\}_{n\in\mathbb{N}}$ in $U_h(gl_\infty)$ was given in [11]. Each basis vector $e_n$ is an appropriate $\mathbb{C}[[h]]$-polynomial in the Chevalley generators. The $\mathbb{C}[[h]]$-span $\hat{U}_h(gl_\infty)$ of the basis is dense in $U_h(gl_\infty)$. It consists of all $\mathbb{C}[[h]]$-polynomials in the Chevalley generators. Extend the domain of definition of $\rho$ on $\hat{U}_h(gl_\infty)$ in a natural way: if $\rho$ has already been defined on $a, b\in\hat{U}_h(gl_\infty)$, then set

$$\rho(\alpha a+\beta b)=\alpha\rho(a)+\beta\rho(b)\quad \rho(ab)=\rho(a)\rho(b)\quad a,b\in\hat{U}_h(gl_\infty)\ \alpha,\beta\in\mathbb{C}[[h]]. \tag{20}$$

$U_h(gl_\infty)$ consists of all elements of the form

$$a=\sum_{i=0}^{\infty}a_ih^i\qquad (a_0,a_1,a_2,\ldots\in\hat{U}_h(gl_\infty)). \tag{21}$$

For any $v\in V(\{M\})$ (see (11)) and $a_i$ from (21) we have

$$\Bigg(\sum_{i=0}^{\infty}\rho(a_i)h^i\Bigg)v=\Bigg(\sum_{i=0}^{\infty}\rho(a_i)h^i\Bigg)\Bigg(\sum_{j=0}^{\infty}v_jh^j\Bigg)=\sum_{n=0}^{\infty}\Bigg(\sum_{m=0}^{n}\rho(a_{n-m})v_m\Bigg)h^n\in V(\{M\}) \tag{22}$$

since

$$\sum_{m=0}^{n} \rho(a_{n-m}) v_m \in V_0(\{M\}). \tag{23}$$

Using (22), extend $\rho$ on $U_h(gl_\infty)$:

$$\rho(a) = \sum_{i=0}^{\infty} \rho(a_i) h^i \in \mathrm{End}\, V(\{M\}) \qquad \forall a \in U_h(gl_\infty). \tag{24}$$

Hence $\rho$ is a well defined map from $U_h(gl_\infty)$ into End $V(\{M\})$,

$$\rho\colon U_h(gl_\infty) \to \mathrm{End}\, V(\{M\}). \tag{25}$$

Let $a \in U_h(gl_\infty)$. Then any neighbourhood $W(\rho(a))$ of $\rho(a)$ contains a basic neighbourhood $\rho(a) + h^n\ \mathrm{End}\, V(\{M\}) \subset W(\rho(a))$. We have in mind the $h$-adic topology both in $U_h(gl_\infty)$ and End $V(\{M\})$. Evidently

$$\rho(a + h^n U_h(gl_\infty)) \subset \rho(a) + h^n\ \mathrm{End}\, V(\{M\}) \subset W(\rho(a)) \qquad \forall a \in U_h(gl_\infty). \tag{26}$$

Therefore $\rho$ is a continuous map. It satisfies (20) for any $a, b \in U_h(gl_\infty)$ and $\alpha, \beta \in \mathbb{C}[[h]]$. Therefore $\rho$ is a $\mathbb{C}[[h]]$-homomorphism of $U_h(gl_\infty)$ in End $V(\{M\})$. We have obtained the following result.

*Proposition 2.* The map (25), acting on the $C$-basis according to (15)–(17), defines a representation of $U_h(gl_\infty)$ in $V(\{M\})$.

Any $U_h(gl_\infty)$-module $V(\{M\})$ is a highest weight module with respect to the 'Borel' subalgebra $N_+$, consisting of all $\mathbb{C}[[h]]$-polynomials of the unity and $\{E_i\}_{i\in\mathbb{Z}}$. The highest weight vector $(\hat{M})$, which by definition satisfies the condition $\rho(N_+)(\hat{M}) = 0$, corresponds to the one from (7) with

$$\hat{M}_{i,2k+\theta-1} = M_i\ \forall k \in \mathbb{N} \qquad \theta = 0, 1\ i \in [-\theta - k + 1, k - 1]. \tag{27}$$

Moreover, $V(\{M\}) = \rho(U_h(gl_\infty))(\hat{M})$. Since $V(\{M\})$ contains no other singular vectors, vectors annihilated from $\rho(N_+)$, each $V(\{M\})$ is an irreducible $U_h(gl_\infty)$-module. The proof of the latter follows from the results in [12] and the observation that each (deformed) matrix element in the transformation relations (15)–(17) is zero only if the corresponding non-deformed matrix element vanishes.

We note in conclusion that all our results remain valid for $h \notin \mathrm{i}\pi\mathbb{Q}$ ($\mathbb{Q}$—all rational numbers), namely in the case $q$ is a number, which is not a root of one.

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